Figure 8. (*a.*) Spores encapsulating and dividing.
(*b.*) A small group of capsules.
(*c.*) A large group of capsules.
(*d.*) A group of capsules surrounded by a common envelope.
(*e.*) Sporules escaping from their capsules.
(*f.*) Large empty capsules.
(*g.*) Phases undergone by a single capsule.
Figure 9. (*a.*) Large finely granular brown sphere.
(*b.*) Smaller coarsely granular spheres.
Figure 10. Patch of naked spores from surface of liquid.
Figure 11. Spores germinating into *Spirilla*, at first comma-shaped.
Figure 12. *Spirilla* resembling germinating spores.
Figure 13. Successive drawings of a portion of same filament, showing extreme irregularity of division of spores.
Figure 14. Irregular division outside filament—(*a*) and (*b*), naked; (*c*) within capsule.

X. "On an Easy and at the same time Accurate Method of Determining the Ratio of the Dispersions of Glasses intended for Objectives." By Professor G. G. STOKES, M.A., Sec. R.S. Received June 18, 1878.

In examining the dispersive powers of a great variety of glasses prepared by the late Rev. W. Vernon Harcourt, I had occasion to examine several prisms which were too much striated to show clearly even the boldest dark lines of the solar spectrum. I found that I was able to get a fair measure of the dispersive powers even of these by a method depending on the achromatizing of one prism by another. If the method succeeded even with such prisms, it stands to reason that it would be still more successful with prisms of good glass.

For the construction of an objective we require but one datum as regards the dispersions, namely, the ratio of the dispersions, or rather, the ratio which on being treated as if it were the ratio of the dispersions gives the best results in practice.

If it were not for irrationality, the matter would be comparatively simple. The ratio of the dispersions would then be the same for whatever interval of the spectrum it were taken; and we should merely have to take two well-defined lines, bright or dark, situated as nearly as may be at the extremities of the spectrum, so that any errors of observation should be divided by as large a quantity as practicable, to measure the refractive indices of the two glasses for each of those two lines, and to take the ratio of the increments in passing from one line to the other. But in consequence of irrationality we get a different ratio according to the particular interval we choose; we are obliged, unless we adopt some different method altogether, to observe more than two lines in each glass; and when we have got the results,

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that is to say, the indices for several fixed lines in each glass, it still becomes a question what it is best to do with them so as to produce the best result.

Fraunhofer proposed an empirical rule for combining the results,* but remarked that the number thus obtained was not exactly that which answered best in practice. I have elsewhere shown reason to think that the rule may be taken to be simply that the ratio of dispersions to be chosen is that for an infinitely small portion of the spectrum at its brightest part; † or in other words, that the focal length of the combination of two glasses must be made a maximum or minimum (it is practically a minimum) at the brightest part of the spectrum.

The refractive index of one glass may be expressed in terms of that of another, or of some standard glass, by an interpolation formula with three, or at most four, terms. The most accurate observations with prisms are only just sufficient to show that a fourth term is needed, and for the practical purpose of the construction of object-glasses, where in consequence of irrationality the ends of the spectrum are sure to be a good deal out of focus, it will be amply sufficient to confine ourselves to three terms. The three coefficients in the interpolation formula may be determined by observing in each glass the indices of refraction for three selected lines, though it is well to observe more than three lines, and combine the results.

The angular extent of the spectrum being but small in practical cases, the necessity of determining three constants by observations made in it requires great delicacy of measurement. Small errors of observation would easily produce an error in the deduced ratio which would be sensible, or even material, in practice.

For the actual construction of an object-glass we require, as has been already remarked, the knowledge of only a single constant relating to the dispersion, not of two ; and if we can find some test whereby to know when the required condition is satisfied, we may dispense with such extreme accuracy in the angular measurements.

Such a test is afforded by the secondary tints, which change with extreme rapidity when the refracting angle of one of the mutually achromatizing prisms, or the position of one of them in azimuth, is altered. If a moderately narrow object with vertical sides, black on a white ground, or white on a black ground, be viewed through opposed prisms, one of crown and one of flint glass, with a small telescope, and the prisms be set to achromatize each other as nearly as may be, it will be found that the slightest touch altering one of the prisms in azimuth alters notably the secondary tints.

The secondary tints in an objective are readily shown by directing the telescope to a vertical line separating light from dark, such as the edge of a chimney seen in the shade against the sky, and covering half the object-glass with a screen having a vertical edge. So delicate is this test that on testing different telescopes by well-known opticians, a difference in the mode of achromatism may be detected. The best results are said to be obtained when the secondary green is intermediate between green and yellow. This corresponds to making the focal length a minimum for the brightest part of the spectrum.

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* "Denkschriften der K. Akad. der Wiss. zu München," vol. v, p. 215.

† "Report of the British Association for 1855," part ii, p. 14.

To enable me to form a judgment as to the sharpness of the test furnished by the tint of the secondary green, as compared with the performance of an object-glass, I tried the following experiment.

A set of parallel lines of increasing fineness was ruled with ink on a sheet of white paper, and a broader black object was laid on it as well, parallel to the lines. The paper was placed, with the black lines vertical, at a considerable distance in a lawn, and was viewed through two opposed prisms, one of crown glass and the other of flint, of such angles as nearly to achromatize each other in the positions of minimum deviation, and then through a small telescope. The achromatism was now effected, and varied in character, by moving one of the prisms slightly in azimuth, and after each alteration the telescope was focused afresh, to get the sharpest vision that could be had. I found that the azimuth of the prism was fixed within decidedly narrower limits by the condition that the secondary green should be of such or such a tint, even though no attempt was made to determine the tint otherwise than by memory, than by the condition that the vision of the fine lines should be as sharp as possible. Now a small element of a double object-glass may be regarded, so far as chromatic compensation is concerned, as a pair of opposed prisms; and therefore we may infer that the tint of the secondary green ought to be at the very least as sharp a test of the goodness of the chromatic compensation as the actual performance of the telescope. And such Mr. Thomas Grubb, to whom I mentioned the test, found to be actually the case in the progress of the construction of the 15-inch refractor for the Royal Society, the instrument at present in the hands of Mr. Huggins.

It follows therefore that two opposed prisms, representing the glasses to be employed in an objective, are to be deemed to achromatize each other when one of the two secondary colours is about midway between yellow and green.

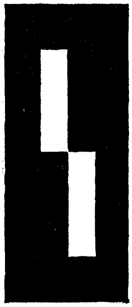
The condition of achromatism of two opposed prisms is given in the ordinary treatises on optics, but, so far as I have seen, rather as an optical curiosity than as a matter of practical utility. Sir David Brewster in his treatise on "New Philosophical Instruments," p. 292, alludes to it as furnishing a conceivable mode of determining dispersive powers, but mentions it only to condemn it. I cannot imagine why, for at least with the modifications which I have been led to introduce in putting it in practice, I find it to be no less excellent than easy.

The experimental arrangements are a good deal simplified by making the prisms to be compared achromatize successively one and the same prism chosen arbitrarily, and retained in a fixed position, instead of making them directly achromatize each other. I will first describe the method as I used it, premising that as the prisms for which it was primarily designed were not of good glass, I was content with less perfect arrangements than would have been desirable for really good glasses. Nevertheless the apparatus when used with good prisms gave very good results. The object observed was a vertical slit, so wide that when viewed through the opposed prisms a broad white stripe was seen, merely fringed at the two edges with the secondary green and purple. The slit was fixed at one end of a horizontal plank, near the other end of which was fixed a vertical lens of about four feet focus. The slit was in the principal focus of the lens. The plank rested on three knobs, one under the slit, the other two near the other end. This mode of support prevented torsion of the plank, which would have produced a lateral derangement of the slit. The rays from the slit, rendered parallel by the lens, fell upon a prism, which I will call the primary prism. This prism is fixed during the observation. Its angle and dispersive power, and the azimuth in which it is set, are arbitrary within wide limits.

The beam emerging from the primary prism falls, at a little distance, on a second prism resting on a stand movable along with the verniers of a protractor reading to minutes (for the loan of which I am indebted to Professor Miller), which latter rests on the plank. As a matter of convenience, the prism has an independent motion in azimuth, but when once placed in a convenient position it is not afterwards moved except along with the verniers.

The beam refracted and dispersed in contrary directions by the two prisms is received on the object-glass of an achromatic telescope unconnected with the graduation, with which the image of the slit, if so wide an aperture may so be called, is viewed. In general the edges of the aperture are seen coloured blue and red, which colours, on turning the prism through its azimuth of achromatism, are changed into red and blue by passing through the secondary green and purple. That edge of the slit at which the green is seen is alone attended to, and is treated as the fiducial edge. The prisms are deemed to achromatize each other when this colour is midway between yellow and green.

Should the observer wish to aid his judgment by observing the purple as well, it would be proper to use an aperture of the form represented in the figure, so that the green and purple should be seen right and left along a common edge, and therefore at the same angle of incidence, or rather at angles of incidence having the same horizontal projection. I do not know that anything would be gained by this; I have found it sufficient to attend to the green.



The light employed had best be that of the sky, reflected horizontally by a looking-glass. It should be fairly bright, but not approaching to dazzling. Thus the light reflected from the sky near the sun would be too bright. The room need not be darkened; in fact, it is better that the eye should be kept fresh for the appreciation of colour by habitually looking about on ordinary objects. A simple collimating lens such as I have described is sufficient, though doubtless an achromatic would theoretically be an improvement. It should be of longish focus, at least in the case of a simple lens, lest any slight displacement right or left of the middle of the incident beam should introduce a minute dispersion due to the lens acting as a prism. If the middle of the beam be not quite central, that does not signify, provided the eccentricity be constant; for then the minute dispersion is merely added to that of the primary prism, which is arbitrary. To ensure constancy of incidence on the lens, it is well to limit it by an aperture with vertical sides, and to take care that the beam employed is wide enough to fill the aperture. Similarly it is well to take care that the beam falls centrically, or pretty fairly so, on the achromatic viewing telescope. But what is of much more consequence is that the ray passing through the optical centre of the object-glass should pass centrically through the eye-piece, as otherwise the dispersion of the eye-piece for eccentrical pencils would alter the secondary tint. It is well, therefore, that the viewing telescope should be furnished with cross wires. The telescope is then moved a little till the fiducial edge is on the cross wires when the tint is observed.

The determination of the azimuth of achromatism is the capital observation on which the accuracy of the result depends; in comparison with it, the rest of the measurements required may be deemed exact. Accordingly a number of observations of this azimuth should be taken, and the mean adopted. The mean error will vary with circumstances, but it may be taken ordinarily as a few minutes.

The mean reading for achromatism by itself alone gives nothing, and must be combined with another determination in order to be available. The second determination which I found it most convenient to make was that of the angle of incidence of a particular part of the spectrum on the second prism for a known azimuth of this prism.

If the brightest part of the spectrum were marked by a definite line, we should choose that. As it is, the line D, though not exactly at the brightest part, lies sufficiently near it for our purpose, as will be better understood presently. No dark lines can be seen in the reflected light, because we are using an aperture, and not a mere slit. But if the slit be illuminated by a soda flame instead of daylight, a well-defined yellow image of the aperture will be seen, the fiducial edge of which can be pointed at with precision.

To determine the angle of incidence I have employed two methods, both of which I think it will be worth while to describe. The first is the less direct, and involves a little subsequent calculation, but has the advantage of not requiring any additional apparatus beyond what is wanted for the determination of the azimuth of achromatism. In carrying out the first, and in applying either, I suppose the angle of the prism and its index for the line D known from a determination in the usual way.

First Method.—The azimuth of minimum deviation for the line D could be determined pretty fairly by bringing the wire of the viewing telescope to the fiducial edge when the aperture is illuminated by a soda flame, and the edge is in or near its stationary position, and taking the mean of several determinations of the azimuth at which the edge is stationary. The angle of incidence for minimum deviation is virtually measured in the process of finding the index, and is therefore known; and by applying with its proper sign the difference of mean readings for achromatism with daylight and for minimum deviation with a soda flame, we get the angle of incidence for achromatism.

This method is mentioned merely as naturally leading up to that actually employed. It must be rejected as too slovenly, since the uncertainty of the determination of the azimuth of minimum deviation is liable to be greater than that of the azimuth of achromatism.

If we place the wire of the viewing telescope some way off the stationary position of the fiducial edge, there are two azimuths of the prism at which the edge will be on the wire, across which it will move with a finite velocity as the prism is moved in azimuth. Hence either azimuth could be determined with accuracy, and if they were equidistant from the azimuth of minimum deviation, the latter could be determined at once. This, however, is not the case, but nevertheless the azimuth of minimum deviation can be determined from the result.

Since the course of light may be reversed in refraction, it readily follows that in passing from one to the other of two azimuths which give equal deviations, the angles of incidence and emergence are simply interchanged. Hence when the prism is turned from one to the other of two azimuths for which the image of the fiducial edge is on the wire of the viewing telescope, the angle moved through is equal to the difference between the angles of incidence and emergence in either position. Both azimuths, and therefore their difference, can be determined with accuracy, provided the azimuths be sufficiently remote from that of minimum deviation.

The process of observation, then, is this. Set the prism a good way, such as 10° or 15°, from the azimuth of minimum deviation, and read the graduation. Turn the viewing telescope till the fiducial edge is on its wire. Taking care to keep the telescope fixed, turn the prism through the position of minimum deviation till the edge is again on the wire, and read again.

Let ϕ, ψ be the angles of incidence and emergence or emergence and incidence, of which let ϕ be the greater. Let $2\,a$ be the angle, i, of the prism, $2\,\beta$ the sum of $2\,a$ and the minimum deviation, $2\,\gamma$ the measured angle through which the prism has been turned, or $\phi-\psi$. Since the sum of the internal angles $=2\,a$, we may represent them by $a+x$ and $a-x$. Let $\phi+\psi$ be denoted by $2\,y$. Then we shall have—

$$\frac{\sin(y+\gamma)}{\sin(a+x)}=\frac{\sin(y-\gamma)}{\sin(a-x)}=\frac{\sin\beta}{\sin a}.$$

Eliminating x from these two equations, we find—

$$\cos^2 y=\frac{\cos^2 a\cos(\beta-\gamma)\cos(\beta+\gamma)}{\cos(a-\gamma)\cos(a+\gamma)}\;\ldots\;(1),$$

which gives y, and then $y\pm\gamma$, *i.e.*, ϕ or ψ, is known. The angle of incidence for a known reading of the graduated circle being known, we have only to apply the difference between this reading and that for achromatism to that angle of incidence in order to get the angle of incidence for the azimuth of achromatism.

Second Method.—In this a little telescope is used, which is attached to the stand of the prism, so as to move with the verniers of the circle. The telescope need not be achromatic, but has cross wires in its principal focus.

After determining the azimuth of achromatism, the slit is illuminated by a soda flame, and the prism with the verniers turned till the fiducial edge is on the wires of the measuring telescope, when the circle is read. The prism is then removed, and the measuring telescope turned till the fiducial edge is seen directly, and the circle is read again. Half the supplement of the difference of the readings gives the angle of incidence when the reflected image was on the cross wires; and by applying the difference of readings for reflection and for achromatism, we get the angle of incidence in the position of achromatism.

This angle, ψ, having been determined by either of the above methods, we have, by the known formula—

$$\frac{d\mu}{\operatorname{cosec} i \cos\phi' \cos\psi} = \frac{d\mu_{,}}{\operatorname{cosec} i_{,} \cos\phi_{,}' \cos\psi_{,}'} \ldots (2),$$

where the letters with the suffix $_{,}$ refer to the second prism. For the prisms would achromatize each other, as is supposed in the deduction of the above formula in treatises on optics, under the same circumstances in which they would achromatize, in succession, the same spectrum. In the application of the formula, it is to be remembered that, of the two angles ϕ, ψ, the former is that which lies on the side of the white light, and is, therefore, the angle of incidence for the first, but of emergence for the second, of two prisms which mutually achromatize each other.

In the application of the formula (2) the angles ϕ', ψ belong, strictly speaking, to the brightest part of the spectrum, which for shortness I will call M, for which the value of the differential coefficient $d\mu_{,} : d\mu$ is supposed to be sought. But the distance of M from D is so small that it will hardly make any sensible error if we use the values of the angles belonging to D, for not only is the correction to the product $\cos\phi' \cos\psi$ for either prism very small, but the two corrections are in the same direction, and therefore tend to neutralise each other in the ratio of the products, with which alone we are concerned. If, however, we care to introduce the correction, it can be done at the expense of a little additional calculation. In a crown glass the index for M may be taken as greater by 0·001 than that for D, and in a flint glass as greater by 0·001 multiplied by a rough value of $d\mu_{,} : d\mu$. The angle of emergence ϕ may be taken to be the same for M as for D. For the deviation, regarded as the function of the index, is a maximum or minimum for M; and D being so near M, the difference of deviations for D and M will be quite insensible compared with the errors of observation of the azimuth of achromatism, with which it is associated. Let the letters $'\mu$, $'\psi$, &c., refer to M, while μ, ψ, &c., refer to D. ψ is obtained by observation, and ψ', ϕ' must, in any case, be calculated from thence by the known values of μ and i. We have now merely to calculate the $'\phi'$ and $'\psi$ for each glass from the formulæ

$$\sin{}'\phi' = \frac{\mu}{{}'\mu}\sin\phi', \quad {}'\psi' = i - {}'\phi', \quad \sin{}'\psi = {}'\mu \sin{}'\psi',$$

and substitute these values in the equation (2) instead of those belonging to the line D.

The primary prism had best be made of very low flint glass (or else be a compound prism formed of two prisms of crown and flint glass respectively, with their angles turned the same way), so as to fall about midway between the glasses to be compared, and thereby divide between them the irrationality which has to be encountered in the observation. The observation of the azimuth of achromatism is most accurate when there is little or no irrationality; and, if preferred, the crown glasses might be compared with a standard crown, and the flint glasses with a standard flint, the primary prism being in the one case any crown glass prism that happens to be at hand, and in the other case a flint glass prism. The crown glass to be measured being compared with the standard crown by using them in succession to achromatize the same primary crown in the same position, and similarly for the flint glass to be measured and the standard flint, we can deduce the ratio of the dispersions of the crown and flint glasses to be measured if we know, once for all, the ratio of the dispersions of the standard crown and flint glasses. This may be determined by a specially careful series of observations of the kind above described, made once for all, or, if preferred, by the method of indices.

The direct comparison of a crown and flint glass is, however, so accurate, especially if a glass of intermediate quality be used for the primary prism, that I feel satisfied it will suffice for practical purposes. It is hardly necessary to observe that, if the primary prism be of intermediate quality to the two compared, the right hand edge of the aperture will be the fiducial edge in the one case, and the left hand edge in the other. In saying this, I assume that we have not extravagant inclinations or differences of angles to deal with, for there is a *quasi*-irrationality observed even when two prisms of the same glass, but of different angles, achromatize each other, which is, however, ordinarily so small that it may be neglected in comparison with the real irrationality of the media.

By turning the primary prism into a different azimuth, or substituting a different primary prism, and repeating the observation, an estimate may be formed of the degree of reliance that may be placed on the results.

To give an idea of the degree of accuracy of which the results are susceptible, I subjoin a few numbers extracted from my note-book. The prisms designated H 74, H 88, H 98, were experimental prisms of phosphatic glass of different compositions. They were more or less striated, but were good enough to show the principal fixed lines of the spectrum. In the different determinations of the ratio of dispersions, the primary prism was set at different azimuths. In the calculation of (2) the indices for D were used. The differences from the means are exhibited.

H 74 to H 98, 1·882, 1·892; mean, 1·887; differences, − 0·005, + 0·005.

H 74 to H 98, 1·755, 1·761, 1·781; mean, 1·766; differences, − 0·011, − 0·005, + 0·015.

It will be seen that, even with prisms such as these, by taking the mean of different determinations, the uncertainty can hardly be as great as one half per cent.

Extremely small prisms are quite sufficient for the determination of the ratio of the dispersions of the glasses by the above method. It may, however, happen that an optician cannot afford to remove even so small a piece of glass from a disk intended for an objective, and has not a specimen of glass on the identity of which with the glass of his disk he can thoroughly rely. In such a case it is necessary to determine the optical constants of the disk by means of facets cut on the disk itself. A heavy and costly disk cannot be treated like a small prism, and mounted on a small graduated instrument in the manner I have supposed a small prism treated. To compare the ratio of the dispersions of two such disks, one of crown glass and the other of flint, the most convenient way would seem to be to leave the disk fixed, let the light pass through it first, and then achromatize it by a small prism of very low flint glass, mounted on a small graduated instrument in the manner already explained. The dispersions of the disks would be compared with each other by comparing them in succession with the same intermediate prism.

This, however, demands an additional determination beyond what was required in the other process, since the prism through which the light first passes is not the same in the two cases. The element which best lends itself to measurement is the angle of incidence on the first surface. The most convenient mode of measuring this must depend on the general disposition of the apparatus adopted to take the measurements for the angle of the disk and the deviation of some one line, which must be made in any case; it is accordingly best left to the choice of the observer.

XI. "On the Reversal of the Lines of Metallic Vapours." By G. D. Liveing, M.A., Professor of Chemistry, and J. Dewar, M.A., F.R.S., Jacksonian Professor, University of Cambridge. No. III. Received June 19, 1878.

In our last communication to the Royal Society we described certain absorption lines, which we had observed to be produced by the vapour of magnesium in the presence of hydrogen, and certain other lines which were observed when potassium, and others when sodium, was present, in addition to magnesium and hydrogen. These lines correspond to no known emission lines of those elements; but, inasmuch as they appeared to be regularly produced by the mixtures